**Chapter 01: Linear Differential Equations**

Table of Contents

[1.1 Classification of Differential Equations 3](#_Toc67051008)

[1.2: Solutions 5](#_Toc67051009)

[Formation of Differential Equations 5](#_Toc67051010)

[1.3: Initial Value Problems, Boundary Value Problems and Existence of Solutions 9](#_Toc67051011)

[Initial Value Problem 9](#_Toc67051012)

[Boundary Value Problem 9](#_Toc67051013)

A differential equation is linear when it is of the first degree in the dependent variable and the derivatives.

A differential equation is linear if:

* Every dependent variable and every variable involved occurs to the first degree only
* No product of dependent variable and its derivative, i.e. ()
* No transcendental function of and its derivative occurs, i.e. ,

## 1.1 Classification of Differential Equations

An equation involving derivatives of one or more dependent variables with respect to one or more independent variables is called a differential equation.

– dependent variable

– independent variable

A partial differential equation (P.D.E.) is an equation involving derivates of one or more dependent variables with respect to more than one independent variables.

- partial differentiation

An ordinary differential equation (O.D.E.) is an equation involving derivates of one or more dependent variables with respect to one independent variable.

The order of a differential equation is the order of the highest derivatives involved.

The degree of a differential equation is the exponent of the highest order derivate when the differential equation is a polynomial of all derivatives.

Order: 2

Degree: 1

Order: 2

Degree: 1

Order: 3

Degree: 3

Order: 2

Degree: Not Defined

Order: 1

Degree: 1

Order: 2

Degree: Not Defined. Can be fixed.

Order: 2

Degree: 3

## 1.2: Solutions

The solution of a differential equation is a function. The solution of is since putting the differentiation of in place of will make the left side equal to the right side. One differential equation can have several solutions

When a function of the curve or the family of curves will satisfy the differential equation, this will be called the solution of the differential equation.

There are 2 types of solutions, general solutions and particular solutions. A general solution has an arbitrary constant. A particular solution has a specific value for the arbitrary solution.

### Formation of Differential Equations

1. Differentiate the equation (of the family of curves) as many times as the number of arbitrary constants.
2. Eliminate the arbitrary constants.
3. The eliminant is the required differential equation.

Exercise:

Form a differential equation from where and are arbitrary constants.

is differentiated with respect to .

The equation is differentiated again with respect to .

This is the required differential equation. It is a 2nd order, 1st degree, non-linear (since there is a product of and its derivative), ordinary differential equation.

Exercise:

Form a differential equation from .

This is the required differential equation. It is a 2nd order, 1st degree, linear, ordinary differential equation.

Form a differential equation of a family of circles which touch the -axis at the origin.

Center:

Radius:

Equation of the circle:

This is the required differential equation. It is a 1st degree, 1st order, non-linear, ordinary differential equation.

Find the differential equation of all straight lines at a unit distance from the origin.

Find the differential equation of the system of ellipses having their axes alone the -axis and -axis.

Show that is a solution of .

Show that is a solution of .

Show that has no real solution.

and are both . So, the equation is always . Thus, the given equation can never have a real solution.

Show that has a one parameter family of solutions of the form , where is an arbitrary constant.

Let .

## 1.3: Initial Value Problems, Boundary Value Problems and Existence of Solutions

### Initial Value Problem

A differential equation that has given conditions allows us to find the specific function that satisfies the differential equation, rather than a family of functions. These types of problems are called initial value problems (IVP).

Defined by or .

### Boundary Value Problem

If conditions are given for more than one point of and the differential equation is order 2 or greater, it is called a boundary value problem (BVP).

Defined by and .

If is a general solution of the differential equation , find the particular solution if and .

a) Show that is a solution of the following initial value problem:

Since all the conditions are met, is a solution to the given problem.

b) Is also a solution of this problem? Explain why or why not.

Since all the conditions are not met, is not a solution to the given problem.